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ABSTRACT

This module is the eighth in a series of 12 learning modules designed to teach occupational mathematics. Blocks of informative material and rules are followed by examples and practice problems. The solutions to the practice problems are found at the end of the module. Specific topics covered include comparison of numbers, pure numbers, denominate numbers, ratios, rates, proportions, and word problems. (YLB)

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Fundamentals of OCCUPATIONAL MATHEMATICS

Module 8

Ratios, Rates, and Proportions

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Module 8--Ratios, Rates, and Proportions

The comparison of two numbers is a common and important calculation performed by mathematics. The manner in which a person is able to compare two numbers depends upon what kind of quantities they represent. The numbers with which technicians work in mathematics are of two broad classifications. Some numbers are called DENOMINATE NUMBERS and the rest are called PURE NUMBERS. Samples from these two types of values are illustrated in the example:

EXAMPLE 1:

Pure Numbers	Denominate Numbers
1.2	1.2 inches
85	85 people
7.53	$7.53 dollar
48	48 teeth
16.3	16.3 gallons
451	451 miles

DENOMINATE NUMBERS are values which consist of a number together with a unit of measure or a unit name. PURE NUMBERS are values which are only a number. Pure numbers have no unit of measure and they do not name the thing which has that quantity.

The denominate numbers which have a unit of measure in Example 1 include the 1.2 inches, $7.53, 16.3 gallons, and 451 miles. The denominate numbers which have a unit name are the 85 people and 48 teeth.

The manner in which two numbers are compared depends upon what kind of numbers they represent. When two denominate numbers have the same unit of measure or have the same unit name, then they are said to be LIKE DENOMINATE NUMBERS.

In LESSON 4 numbers were compared using subtraction. Subtraction is used to indicate how much more one number is than the other. Subtraction can be used to compare two numbers only if both numbers are pure numbers or if both numbers are like denominate numbers. One can not subtract 48 teeth from 85 people.

LESSON 8 is about how to compare two numbers using fractions. This process is similar to the comparison by division performed in LESSON 6. Here the result is called a ratio or a rate.

A RATIO is a comparison, by division in a specific order, of two pure numbers or two like denominate numbers.

The ratio of the first number n to a second number m can be written in either of three forms:

(1) with numbers/word as "n to m",

(2) as fraction n/m , or

(3) with colon as n:m.

When it is known that the fraction n is a ratio, then the ratio n/m is read as "n to m". The ratio n:m as "n to m".

EXAMPLE 2: If the length of a rectangle is 18 inches and the width is 5 inches, what is the length to width ratio?

Solution: The following observations need to be made:

The first number is length = 18 inches.
The second number is width = 5 inches.
They are LIKE denominate numbers.
Length to width ratio = $\frac{18 \text{ inches}}{5 \text{ inches}}$

$$= \frac{18}{5}$$

The answer is any of forms $\frac{18}{5}$ = 18:5 = 18 to 5.

Note the cancellation of the like unit of measure; inches.

EXAMPLE 3: If there are 8 pounds of tin and 3 pounds of copper in an alloy, what is the ratio of copper to tin?

Solution: Make the following observations:

The first number is the copper amount, 3 lb.
The second number is the tin amount, 8 lb.
They are like denominate numbers.
Copper to tin ratio = 3 lb. to 8 lb.

$$= \frac{3 \text{ lb.}}{8 \text{ lb.}} = \frac{3}{8}$$

The answer is 3:8 or 3 to 8 or $\frac{3}{8}$

When ratios are used, they are usually reduced to lowest terms. To reduce a ratio to lowest terms, write the ratio as a proper fraction and use the fraction reducing skills learned in LESSON 7. If the ratio should reduce to a whole number, write "1" as the denominator to make it clear that the numbers are being compared by ratio.

EXAMPLE 4: State the comparison of 72 with 42 as a ratio in lowest terms.

Solution: Observe that the order is:

72 is the first number,
42 is the second number, and these amounts are pure numbers.

$$\text{Ratio} = \frac{72}{42} = \frac{72 \div 6}{42 \div 6} = \frac{12}{7}$$

EXAMPLE 5: Steel can be worked in a lathe at a cutting speed of 25 ft/min. Stainless steel can be worked in the lathe at a cutting speed of 15 ft/min. What is the ratio of the cutting speeds of stainless steel to ordinary steel?

Solution: Observe that:

Cutting speed of stainless steel is first:
15 ft/min
Cutting speed of ordinary steel is second:
25 ft/min
These speeds are like denominate numbers.

$$\text{Ratio of cutting speeds} = \frac{15 \text{ ft/min}}{25 \text{ ft/min}} = \frac{15}{25} = \frac{3}{5}$$

Answer: $\frac{3}{5} = 3:5 = 3$ to 5

EXAMPLE 6: Two pulleys are belted so that there is no slippage. The driver pulley has a diameter of 2 ft 3 in. and the driven pulley has a diameter of 9 in. The ratio of the diameter of the driver pulley to the diameter of the driven pulley is called the mechanical advantage, MA. What is the mechanical advantage of this pair of pulleys?

Solution: Observe that:

First number is driver pulley diameter, 2 ft 3 in.
Second number is driven pulley diameter, 9 in.
These are not like denominate numbers.
Change to the same unit of measure, inch.

$$2 \text{ ft } 3 \text{ in} = 2 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} + 3 \text{ in}$$

$$= 24 \text{ in} + 3 \text{ in}$$

$$= 27 \text{ in}$$

$$\text{Mechanical advantage} = \frac{27 \text{ in}}{9 \text{ in}} = \frac{27}{9} = \frac{3}{1}$$

Answer: $MA = \frac{3}{1} = 3:1 = 3$ to 1

PRACTICE PROBLEMS: Write each of the following as a ratio reduced to lowest terms.

1. 30 inches to 63 inches
2. 25 grams to 15 grams
3. 48 mm to 32 mm
4. 2 ft 8 in to 4 in
5. 6 in to 3 ft 3 in
6. A bearing bronz mix includes 96 lb of copper and 15 lb of lead. Find the ratio of copper to lead.
7. A flywheel has 72 teeth and a starter drive-gear has 15 teeth. Find the ratio of flywheel teeth to drive-gear teeth.
8. A transformer has 45 turns in the primary coil and 540 turns in the secondary coil. Find the ratio of secondary turns to primary turns.
9. What is the alternator to engine-drive ratio if the alternator turns at 1125 rpm when the engine is idling at 500 rpm?
10. A structure has 3290 ft^2 of wall area, excluding openings, and 1880 ft^2 of window area. Find the ratio of wall area to window area.

Ratios which are expressed in the form of some number to 1 are desirable in several situations. The ratio of a number to 1 permits a quick comparison of their relative sizes. The driver pulley in Example 6 has a diameter that is 3 times as long as that of the driven pulley. When the numbers to be compared include fractions or decimals, then the reduction of n:m is somewhere between awkward and difficult. Computing their ratio as a number to 1 is usually acceptable.

Application problems which express some type of efficiency by using an input to output ratio, like mechanical advantage, are often written as the ratio of a number to 1.

To change the ratio n:m into a ratio to 1, divide n by m. That quotient is used to write the ratio $(n \div m):1$. For the ratio to 1 problems, when $n \div m$ produces a decimal of many decimal digits, round-off the $n \div m$ value to the nearest hundredth.

EXAMPLE 7: Write each of the following ratios as a ratio to 1.

(a) 103.8 mm to 17.3 mm
(b) 2 3/16 in to 5/8 in
(c) 4 1/4 in to 1 15/16 in

Solution:

(a) $(103.8\ \text{mm}) \div (17.3\ \text{mm}) = (103.8 \div 17.3)(\text{mm} \div \text{mm})$
$= 6$

Answer is ratio 6:1.

(b) Convert both fractions to decimal form.
$2\ 3/16 = (3 \div 16) + 2$

$= 0.1875 + 2$

$= 2.1875$

and

$5/8 = 5 \div 8$

$= 0.625$

$2\ 3/16\ \text{in} \div 5/8\ \text{in} = (2.1875\ \text{in}) \div (0.625\ \text{in})$

$= (2.1875 \div 0.625)(\text{in} \div \text{in})$

$= 3.5$

Answer is ratio 3.5:1

(c) Convert both fractions to decimal form.
$4\ 1/4 = (1 \div 4) + 4$

$= 4.25$

and

$1\ 15/16 = (15 \div 16) + 1$

$= 1.9375$

$4\ 1/4\ \text{in} \div 1\ 15/16\ \text{in} = (4.25 \div 1.9375)(\text{in} \div \text{in})$

$= 2.193548387$

≈ 2.19

Answer is ratio 2.19:1

EXAMPLE 8: When two gears are meshed, the ratio of the number of teeth in the driving gear to the number of teeth in the driven gear is called the gear ratio. A driving gear with 68 teeth is meshed with a driven gear with 24 teeth. Compute this gear ratio in the form of a number to 1.

Solution: Identify that:

First amount is the driving gear size; 68 teeth.
Second amount is the driven gear size; 24 teeth.
These are like denominate numbers.

Gear ratio = (68 teeth)÷(24 teeth):1

= (68 ÷ 24):1

= 2.83333333:1

≈ 2.83:1

EXAMPLE 9: A drive shaft with a rotational speed of 825 rpm drives a second shaft at a rotational speed of 543 rpm. The speed ratio is the ratio of the rotational speed of the drive shaft to the rotational speed of the second shaft. What is the speed ratio of these shafts?

Solution: Identify that:

First quantity is drive shaft speed; 825 rpm.
Second quantity is second shaft speed; 543 rpm.
These are like denominate numbers.

Speed ratio = (825 rpm)÷(543 rpm):1
= (825 ÷ 543):1
= 1.519337017 :1
≈ 1.52:1

PRACTICE PROBLEMS: Compute each of the following ratios and express the result as the ratio of a number to 1. Round to the nearest hundredth as needed.

11. 45 to 160

12. 38.9 mm to 17.3 mm

13. 18.5 mm to 36.2 mm

14. 10 1/4 to 2 1/2

15. A driving gear with 42 teeth is meshed with a driven gear with 98 teeth. Compute this gear ratio in the form of a number to 1.

16. A drive shaft with a rotational speed of 750 rpm drives a second shaft at a rotational speed of 215 rpm. What is the speed ratio of these shafts?

17. What is the alternator to engine-drive ratio as a number to 1 if the alternator turns at 1125 rpm when the engine is idling at 500 rpm?

18. The factor of safety is the ratio of the ultimate stress of a bar to the actual unit stress that exists in a bar. Find the factor of safety of a copper casting under compression if the unit stress is 7500 pounds and the ultimate stress is 40,000 pounds.

19. The rear-axle ratio of a car is the ratio of the number of teeth on the ring gear to the number of teeth on the pinion gear. Compute the rear-axle ratio as a number to 1 when the ring gear has 44 teeth and the pinion gear has 12.

20. The compression ratio of an engine is the ratio of the maximum space in the cylinder to the minimum space in the cylinder. Find the compression ratio as a number to 1 if the maximum space is 357cc and the minimum space is 42cc.

In a ratio, either two pure numbers or two like denominate numbers are compared. A ratio which has been reduced to lowest terms will no longer have a unit of measure. The simplified ratio of n:m is a pair of unitless numbers.

Several common situations require a comparison by division of two denominate numbers of UNLIKE units. The result is somewhat similar to the ratio of a number to 1. One important new feature will be that the unlike units will not cancel each other. A situation familiar to most adults is that of the calculation of gasoline consumption.

EXAMPLE 10: Find the efficiency rating of an automobile which used 16.8 gallons of gasoline on a trip of 378 miles.

Solution: This is similar to the ratio of miles to gallons.

$$\text{Consumption} = \frac{378 \text{ miles}}{16.8 \text{ gallons}}$$

$$= (378 \div 16.8)(\text{miles} \div \text{gallons})$$

$$= 22.5 \ (\text{miles} \div \text{gallon})$$

The unit of measure for this rate is further abbreviated as:
Consumption = 22.5 miles/gallon
= 22.5 miles per gallon
= 22.5 mpg

Notice that this consumption rate is not written as a ratio. The units of measure expressions miles/gallon, miles per gallon and mpg all mean the same thing.

A RATE is the comparison, by division in a specific order, of two unlike denominate numbers whose units do not cancel.

EXAMPLE 11: Find the speed of a pulley which makes 250 full turns in 3 minutes.

Solution: Consider the following information:

250 full turns equals 250 revolutions.

RATE of speed is measured in revolutions per minute = rpm.

The first number in rpm is revolutions.
The second number in rpm is measured in minutes.

$$\text{Rate of speed} = \frac{250 \text{ revolutions}}{3 \text{ minutes}}$$

$$= (250 \div 3)(\text{rev} \div \text{min})$$

$$= 83.33333333 \text{ rev per min}$$

$$\approx 83.33 \text{ rpm}$$

EXAMPLE 12: A 2 1/2 inch bolt shaft contains 40 threads. What is the rate of threads per inch?

Solution: Observe the following about "threads per inch":

First quantity is measured in threads.
Second quantity is measured in inches.

$$\text{Rate} = \frac{40 \text{ threads}}{2\ 1/2 \text{ inches}}$$

$$= (40 \div 2\ 1/2)(\text{threads} \div \text{inch})$$
$$= (40 \div 2.5) \text{ threads per inch}$$
$$= 16 \text{ threads per inch.}$$

PRACTICE PROBLEMS: Calculate the RATE specified in each problem:

21. Suppose that 17 gallons of oil flow through a feeder pipe in 5 minutes. Find the rate of flow in gallons per minute.

22. If 16 feet of copper tubing costs $13.60, how much does the tubing cost per foot?

23. If there are 55 pounds of pressurized oxygen in a volume of 2.4 ft^3, what is the weight per ft^3 at that pressure?

24. John can make 25 parts in 1 hour. Express his rate of work in parts per minute.

25. John can make 25 parts in 1 hour. Write his task completion rate in minutes per part.

A PROPORTION is an equation that is obtained by setting two RATIOS equal to each other.

Examples of proportions include the equal ratios 5/8 = 20/32, 7:2 = 42:12 and 36 to 3 = 12 to 1. Each proportion contains four numbers called terms. The positions of the terms of a proportion are named in the same order as you would read the numbers. A proportion which uses the fraction ratio forms would have its terms named as $\frac{\text{first term}}{\text{second term}} = \frac{\text{third term}}{\text{fourth term}}$. Proportion 5/8 = 20/32 has its term locations described by saying:

The first term is 5.

The second term is 8.

The third term is 20.

The fourth term is 32.

In the proportion 7:2 = 42:12, the third term is 42 and the second term is 2.

The first and fourth terms are called the EXTREMES of the proportion. The second and third terms are called the MEANS of the proportion. The proportion 36 to 3 = 12 to 1 has the numbers 36 and 1 as the extremes and the numbers 3 and 12 as the means.

In any proportion, the product of the means equals the product of the extremes.

The truth of proportion 36 to 3 = 12 to 1 can be tested by showing that (3)(12) = (36)(1) is true since both equal 36.

EXAMPLE 13: Label each of the following proportions as true or false.

(a) 6/15 = 14/35

(b) 6:4 = 23:16

(c) 5 1/4 to 2 = 21 to 8

Solution (a):

The means are 15 and 14.
The extremes are 6 and 35.

(15)(14) = 210 (6)(35) = 210

The proportion is true.

Solution (b):

The means are 4 and 23.
The extremes are 6 and 16.

(4)(23) = 92 (6)(16) = 96

The proportion is false.

Solution (c):

The means are 2 and 21.
The extremes are 5 1/4 and 8.
(2)(21) = 42
(5 1/4)(8) = (5.25)(8) = 42

The proportion is true.

PRACTICE PROBLEMS:

26. For proportion 6/14 = 15/35 identify
    (a) the second term.
    (b) the fourth term.
    (c) the extreme terms.

27. Given proportion 22 to 6 = 33 to 9, identify
    (a) the third term.
    (b) the extreme terms.
    (c) the mean terms.

Test the truth of the proportion statements in problems 28 through 30.

28. $\frac{3}{10} = \frac{12}{40}$ 29. $\frac{4}{5} = \frac{6}{9}$ 30. $\frac{1.2}{0.5} = \frac{4.2}{1.75}$

If three of the numbers in a proportion are known, then the misssing term to produce a true proportion can always be found. This process of finding the missing term is called SOLVING THE PROPORTION.

EXAMPLE 14: Solve the proportion 3/4 = n/12.

Solution: Calculate the value of n that makes the proportion true.
The means are 4 and n.
The extremes are 3 and 12.
(4)(n) = (3)(12)
(4)(n) = 36
n = 36 ÷ 4
n = 9

Proof: (4)(9) = (3)(12)
36 = 36

EXAMPLE 15: Solve the proportion 12:8 = 11.1:d.

Solution: Find the value of d.
The means are 8 and 11.1.
The extremes are 12 and d.
(8)(11.1) = (12)(d)
88.8 = (12)(d) switch sides so d is on the left
12)(d) = 88.8
d = 88.8 ÷ 12
d = 7.4

Proof: (8)(11.1) = (12)(7.4)
88.8 = 88.8

EXAMPLE 16: Solve the proportion $\frac{149}{x} = \frac{124}{67}$.

Solution: Find the value of x.
(x)(124) = (149)(67)
(x)(124) = 9983
x = 9983 ÷ 124
x = 80.50806452
x ≈ 80.51 rounded to four signf. digits.

Proof: (80.51)(124) ≈ (149)(67)
9983.24 ≈ 9983
Notice that if 9983.24 is rounded to four significant digits, the result would be exactly equal to 9983.

PRACTICE PROBLEMS: Solve the following proportions. Round any extremely long decimals to four significant digits.

31. $\frac{x}{4} = \frac{9}{12}$

32. $\frac{10}{15} = \frac{y}{75}$

33. $\frac{1.1}{6} = \frac{44}{x}$

34. $\frac{x}{9} = \frac{12}{7}$

35. $\frac{0.25}{n} = \frac{8}{48}$

36. $\frac{17}{28} = \frac{153}{d}$

37. $\frac{1}{0.0004} = \frac{700}{B}$

38. $\frac{56}{72} = \frac{T}{48}$

39. $\frac{472}{x} = \frac{793}{64.2}$

40. $\frac{y}{19.3} = \frac{94.7}{6.72}$

41. $\frac{19.6}{3.87} = \frac{n}{4.2}$

42. $\frac{0.120}{x} = \frac{0.575}{277}$

43. $\frac{14.4}{8.4} = \frac{y}{12}$

44. $\frac{26.9}{C} = \frac{0.0417}{0.355}$

45. $\frac{36.9}{104} = \frac{3210}{n}$

46. To find the gear to cut a required number of threads per inch, the proportion C/N = S/L is used. In the formula, C = lathe screw constant, N = number of threads per inch, S = number of teeth in the gear on spindle stud and L = number of teeth in the gear on lead screw. Find the number of teeth in the gear in lead screw if the lathe screw constant is 4, the number of threads per inch is 16, and the number of teeth in the gear on spindle stud is 24.

Some application problems are solved quite easily by proportion. There are two types of proportion problems, direct and inverse, that will need to be recognized. To understand the difference between a direct proportion and an inverse proportion, one must first realize that when working any proportion problem, two variable numbers are always being compared. The two variables of a proportion may react to each other in either of two different ways. The following situations illustrate how the two variables of a proportion may behave in relation to each other.

ILLUSTRATION 1: It takes 300 minutes to produce 15 machined parts of a certain type. How long would it take to make 20 parts?

In Illustration 1, an INCREASE in the number of parts from 15 to 20 will result in an INCREASE in the number of minutes required for production. The variables both change in the same direction, they increase. This type of proportion is called a DIRECT proportion.

ILLUSTRATION 2: If 3 people can produce the parts for a machine in 100 minutes, how long will it take for 5 people to produce the parts?

In Illustration 2, an INCREASE in the number of production people will mean a DECREASE in the number of minutes required to produce the parts. The variables change in opposite directions. This is a type of proportion called an INVERSE proportion.

Let x and y stand for the two related variables of a proportion problem. Measurements of variables x and y from the first case will be called x_1 and y_1. The measurement of x and y from the second case be called x_2 and y_2.

A DIRECT proportion between variable x and variable y occurs when

(a) an increase in x means an increase in y or

(b) a decrease in x means a decrease in y.

The number values of a direct proportion problem are put into a proportion so that the measurements taken in the first case are the first and third terms while the measurements taken in the second case are the second and fourth terms.

DIRECT PROPORTION $$\frac{x_1}{x_2} = \frac{y_1}{y_2}$$

When a direct proportion has been correctly written, the fractions will have:

(1) the two measurements taken during each case placed directly across from each other.

(2) the two terms of a ratio will have the same unit of measure.

(3) none of the fractions are rates where the units will not cancel.

EXAMPLE 17: It takes 300 minutes to produce 15 machined parts of a certain type. How long would it take to make 20 parts?

Solution: Increase in number of parts means an increase in time. Required proportion is DIRECT.

$$\frac{300 \text{ minutes}}{x \text{ minutes}} = \frac{15 \text{ parts}}{25 \text{ parts}}$$

The units of measure cancel to make

$$\frac{300}{x} = \frac{15}{25}$$

$$x)(15) = (300)(25)$$

$$x)(15) = 7500$$

$$x = 7500 \div 15$$

$$x = 500$$

Answer is 500 minutes.

EXAMPLE 18: If an architectural drawing is scaled so that 0.75 inch represents 20 feet, what length represents 5 feet?

Solution: A decrease in actual distance means a decrease in the scale distance. Required proportion is DIRECT.

$$\frac{0.75 \text{ inch}}{x \text{ inch}} = \frac{20 \text{ feet}}{5 \text{ feet}}$$

$$\frac{0.75}{x} = \frac{20}{5}$$

$$(x)(20) = (0.75)(5)$$

$$(x)(20) = 3.75$$

$$x = 3.75 \div 20$$

$$x = 0.1875$$

Answer is 0.1875 inch.

An INVERSE proportion between variable x and variable y occurs when:

(a) an increase in x means a decrease in y or

(b) a decrease in x means an increase in y.

The number values of an inverse proportion problem are put into a proportion so that the measurements taken in the first case are the extremes (first and fourth terms) while the measurements taken in the second case are the means (second and third terms).

INVERSE PROPORTION $\quad \dfrac{x_1}{x_2} = \dfrac{y_2}{y_1}$

When an inverse proportion is correctly written, the fractions will have:

(1) the two measurements taken during a case are placed diagonally across from each other.

(2) the two terms of a ratio have the same unit of measure.

(3) each fraction is a ratio where the units will cancel.

EXAMPLE 19: If 3 people can produce the parts for a machine in 100 minutes, how long will it take for 5 people to produce the parts?

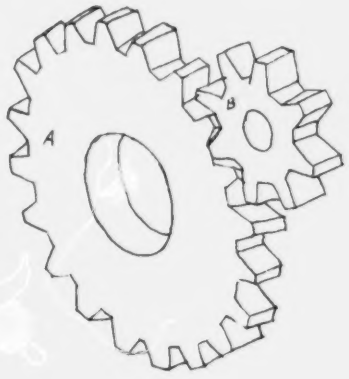
Solution: An increase in the number of working people will mean a decrease in the time to make the parts. Required proportion is INVERSE.

$$\frac{3 \text{ people}}{5 \text{ people}} = \frac{x \text{ minutes}}{100 \text{ minutes}}$$

$$\begin{aligned} 3/5 &= x/100 \\ (100) &= (5)(x) \\ 300 &= (5)(x) \\ (5)(x) &= 300 \\ x &= 300 \div 5 \\ x &= 60 \end{aligned}$$

Answer is 60 minutes.

Two gears, A and B, are meshed as shown on the right. Gear A, which has the most teeth, will make fewer revolutions than gear B. That means that an increase in the number of teeth has resulted in a decrease in the speed of the revolutions. The number of teeth and the speed of a gear are inversely proportional.



EXAMPLE 20: Two gears are in mesh. The driver gear has 20 teeth and makes 50 revolutions per minute. The driven gear has 8 teeth. Find the speed of the driven gear.

Solution: Speed and number of teeth of meshed gears are related by an inverse proportion.

$$\frac{20 \text{ teeth}}{8 \text{ teeth}} = \frac{x \text{ revolutions per minute}}{50 \text{ revolutions per minute}}$$

$$20/8 = x/50$$
$$(8)(x) = (20)(50)$$
$$(8)(x) = 1000$$
$$x = 1000 \div 8$$
$$x = 125$$

Answer is 125 rpm (revolutions per minute)

The key to selecting the correct type of proportion is to ask the question:

If this variable increases in size, how will the other variable change?

Direct proportion variables change in the same direction while inverse proportion variables change in opposite directions.

PRACTICE PROBLEMS: Use direct or inverse proportions to solve the problems.

47. If 0.25 inch on a drawing represents 2 feet, how many feet would a 3 inch length represent?

48. If a car used 12.7 gallons of gas to travel 318 miles, how many gallons would it use to travel 500 miles?

49. A gear with 12 teeth has the speed of 450 revolutions per minute. An 8 tooth gear is in mesh with the 12 tooth gear. What is the speed of the 8 tooth gear?

50. It takes 120 minutes (2 hours) to drive to Omaha at 50 mph. How long would it take to drive to Omaha if the speed limit was 65 mph?

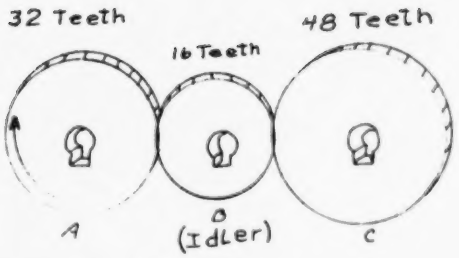
51. It takes four 5-inch pipes 2.25 hours to empty a tank. How long would it take three 5-inch pipes to empty the same tank?

52. Three welders can fabricate 100 braces in a day. How many braces could 5 welders make in one day?

53. George can paint 65 panels in 2 hours. How many panels could he paint in 7 hours?

54. If Jane's car will travel 118 miles on 5 gallons, how many miles can she travel on an 18 gallon full tank?

Use the diagram at the right for problems 55 through 58.



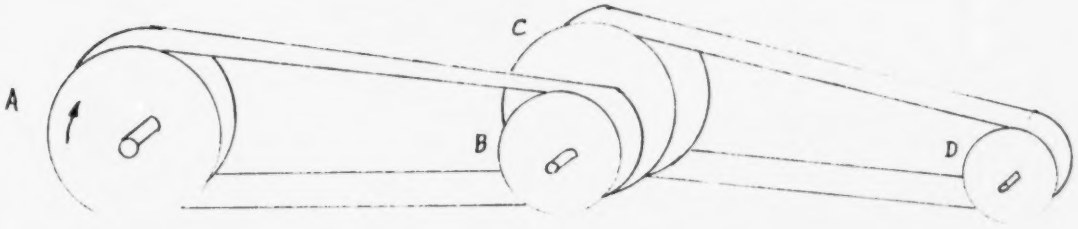
55. Gear A turns at 300 rpm. Find the speed of gear B.

56. Use the speed of gear B from problem 55 to find the speed of gear C.

57. If gear C were to mesh with gear A without the idler gear B,
    (a) what would be the speed of gear C?
    (b) Does the idler cause any change in speed for gear C?

58. Gear A rotates clockwise as shown by the arrow drawn on it.
    (a) In what direction does gear C rotate?
    (b) If gear C were to mesh with gear A without the idler, and A rotates clockwise, then in what direction would gear C rotate?

When two pulleys are attached to the same shaft, they turn at the same speed as is the case with pulleys B and C below. The speed of pulley A is 800 rpm. The diameters of Pulleys A, B, C and D are 100 mm, 25 mm, 87.5 mm and 35 mm, respectively. Use this information and the diagram below for problems 59 through 61.



59. What is the speed of pulley C?

60. What is the speed of pulley D?

61. If pulley A turns clockwise, what direction does each of the other pulleys turn?

PRACTICE PROBLEM SOLUTIONS--Module 8

1. $\frac{10}{21}$
2. $\frac{5}{3}$
3. $\frac{3}{2}$
4. $\frac{8}{1}$
5. $\frac{2}{13}$
6. $\frac{32}{5}$
7. $\frac{24}{5}$
8. $\frac{12}{1}$
9. $\frac{9}{4}$
10. $\frac{7}{4}$
11. 0.28:1
12. 2.25:1
13. 0.51:1
14. 4.1:1
15. 0.43:1
16. 3.49:1
17. 2.25:1
18. 5.33:1
19. 3.67:1
20. 8.5:1
21. 3.4 gal per min
22. $0.85 per ft
23. 23 lb per ft_3
24. .42 parts per min
25. 2.4 parts per min
26. a) 14
    b) 35
    c) 6 and 35
27. a) 33
    b) 22 and 9
    c) 6 and 33
28. True
    120=120
29. False
    36=30
30. True
    2.1=2.1
31. x=3
32. y=50
33. x=240
34. x=15.43
35. n=1.5
36. d=252
37. B=0.28
38. T=37.33
39. x=38.21
40. y=272.0
41. n=21.27
42. x=57.81
43. y=20.57
44. C=229.0
45. n=9047
46. 96 teeth
47. 24 ft
48. 20.0 gal
49. 675 rpm
50. 92.3 min
51. 3 hr
52. 167 braces
53. 227.5 panels
54. 424.8 mi
55. 600 rpm
56. 200 rpm
57. a) 200 rpm
    b) no
58. a) clockwise
    b) counter-clockwise
59. 3200 rpm
60. 8000 rpm
61. all pulleys turn clockwise